THE GENERALIZED METHOD OF CHARACTERISTICS FOR WAVEFORM RELAXATION ANALYSIS OF LOSSY COUPLED TRANSMISSION LINES

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Abstract

Transient response of lossy coupled transmission lines is simulated by iterative waveform relaxation analyses of equivalent disjoint networks constructed with congruence transformers, FFT waveform generators and characteristic impedances synthesized by the Pade approximation. A phenomenal two order reduction of CPU time and one order savings in computer memory have been achieved. A lossy directional coupler is simulated for illustration.

INTRODUCTION

The method of characteristics(MC) and the method of waveform relaxation(WR) are two seemingly unrelated computational algorithms being developed to improve the efficiency of computer aided analysis of large scale electrical circuits. The MC (1) is a technique for solving partial differential equations (PDE) by transformation of PDEs into ordinary differential equations (ODE) along the characteristic directions whereas the WR method(2) is a technique for solving systems of ODEs by iteration and system decomposition. The MC was introduced by Branin (3) for the transient analysis of ideal transmission line. His idea was subsequently generalized (4) and implemented in circuit simulators for the analysis of coupled transmission lines. Since the MC was conceived for the solution of PDEs (5), other CAD applications of the algorithm are also specialized for the discrete-time simulation of distributed parameter networks (6)-(8). At the other extreme of development of CAD tools, the WR method was tailored for the waveform simulation of lumped-parameter systems of which the MOS integrated circuits(IC) have received most of the attention(2),(9)-(10).

It is the purpose of this paper to show that by generalizing the method of characteristics for waveform relaxation analysis, time domain simulations of lumped-parameter networks interconnected with coupled transmission lines can be carried out more efficiently. The generalized method of characteristics(GMC) has been implemented in ICD (11) on an experimental basis. In comparison to the classsical discrete-time simulation, a phenomenal two order reduction in computer simulation time and one order savings in computer memory requirements have been achieved by applying the GMC.

THEORY

In this first of a sequence of papers devoted to the waveform relaxation anlalysis of coupled transmission lines, we focus our attention on an n-conductor lossy shielded stripline system, which is surrounded by a homogeneous, leakage-free dielectric medium. By applying the congruence transformation technique (4) for an n-conductor system, we obtain an equivalent circuit consisting of n decoupled lossy transmission lines interconnected with congruence transformers as shown in Fig.1. To derive the waveform relaxation algorithm, we assume that the coupled transmission lines are terminated with Thevenin's sources and impedances as shown in Fig.2(a). Thus, from the equivalent circuit we obtain the terminal voltages:

$$V(0)=(1_n-P)^{-1}U_A=U_A+PU_A+P^2U_A+P^3U_A+\ldots (a)$$

$$V(\ell)=(1_n-Q)^{-1}U_B=U_B+QU_B+Q^2U_B+Q^3U_B+\ldots (b)$$

where the delay-matrices P,Q and the equivalent voltage sources U_A,U_B are defined in Table 1. The infinite series of incident and reflected waves given in (a) and (b) are iteratively generated from simulations performed on the disjoint networks shown in Fig.2(b). The lossy characteristic impedances in the disjoint networks are synthesized by the RC ladder networks derived form the Pade approximation (12). The voltage sources connecting in series with the characteristic impedances are obtained by

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the Fast Fourier Transform (FFT). The congruence transformers are simulated by using dependent voltage and current sources. Since the terminal voltages at the opposite ends of the coupled lines are simulated sequentially, the iteration scheme is categorized as the Gauss-Seidel type (2). At the kth iteration, the simulated terminal voltages:

$$V_A^k(s)=(1_n+P+P^2+P^3+...,+P^{k-2})U_A+P^{k-1}V_A^1(s)$$
$$=(1_n-P^{k-1})(1_n-P)^{-1}U_A+P^{k-1}V_A^1(s)$$

$$V_B^k(s)=(1_n+Q+Q^2+Q^3+...,+Q^{k-2})U_B+Q^{k-1}V_B^1(s)$$
$$=(1_n-Q^{k-1})(1_n-Q)^{-1}U_B+Q^{k-1}V_B^1(s)$$

are identical to those of (a) and (b) except for the residue terms $P^{k-1}V_A^1$, $Q^{k-1}V_B^1$ generated from the initial condition of the coupled transmission-line system. For k approaching infinity, the residue terms vanish if the eigenvalues of the delay-matrices P and Q are confined in the left-half s-plane. Terminating the coupled-lines with the Thevenin's equivalent circuits is merely an assumption made for the convenience of deriving the waveform relaxation algorithm. Once the algorithm is established, it can be used for the transient analysis of coupled-lines terminated in active, nonlinear loads. An example is given below for illustration.

WAVEFORM RELAXATION ANALYSIS OF A LOSSY DIRECTIONAL COUPLER

Tabulated in Table 2 are the parameters of the three-conductor coupled-line system,which is driven by a bipolar emitter coupled logic (ECL) gate as shown in Fig.3(a). The coupled-line system is properly terminated such that it would function as a directional coupler (13) if the conductors were lossless. Specifically, the in-phase output of the ECL-gate would be transmitted to the far end of the center conductor without distortion in waveshape. In other words, the out-of-phase output of the ECL-gate would not be coupled from the active outer conductor(#1) to the receiving end of the center conductor. In view of the simulated waveforms shown in Fig.3(c),we observe that such a directional property of signal transmission is remarkably preserved even with the presence of the conductor loss. The waveforms in Fig.3 are obtained by both discrete-time and waveform relaxation analyses. In applying the classical method of characteristics for the discrete-time simulation (4), the coupled lossy conductor system is treated as many segments of coupled lossless lines connected in series with discrete resistors. Such a brute-force approach requires excessive computer memory (2 megabytes) for simulation of the distributed nature of the conductor loss. Furthermore, the computer simulation time is extremely long (IBM 3090 CPU=693 seconds) since the integration time-step is kept smaller than the delay time of the individual lossless-line segment for the entire simulation process. Otherwise, incorrect results characterized with false ringing are produced. In comparison, the waveform relaxation analysis requires much smaller storage (233 kbytes) and a shorter computer time (IBM 3090 CPU=11.07 seconds). As shown in Fig.3, three iterations are sufficient to obtain the convergent solution. Notice that in the second and third iterations, circuit simulation needs not be carried out for the entire time period since the waveforms have already partially converged. Taking advantage of such a latency property of the waveform relaxation, the computer simulation can be further reduced by another factor of two. We use the directional coupler as an example to demonstrate the accuracy of the waveform relaxation method. As shown in Fig.3(d), the signal received at the far-end of the quiet-conductor(#3) is the result of the noise cross-coupled from the active outer-conductor(#1), therefore it has a peak amplitude 10 times smaller than that observed at the far-end of the center-conductor. Here, the root mean square difference betweeen the waveforms obtained by the waveform relaxation and the discrete-time simulation is measured in the tenths of millivolt. The extreme accuracy of the waveform relaxation is achieved by reducing the spectrum aliasing effect in the FFT and increasing the resolution of the spectrum sampling.

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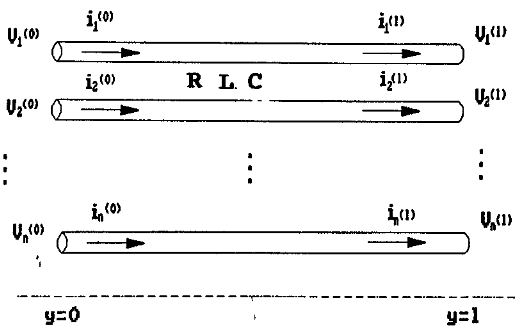

Fig.1(a) An n-conductor lossy coupled transmission-line system.

Table 1. Parameters Defined in the Waveform Relaxation Algorithm

$$P=(1_n+\rho_A)\phi\rho_B\phi\rho_A(1_n+\rho_A)^{-1}$$

$$Q=(1_n+\rho_B)\phi\rho_A\phi\rho_B(1_n+\rho_B)^{-1}$$

$$2U_A=(1_n+P\rho_A^{-1})(1_n-\rho_A)E_A+(1_n+\rho_A)\phi(1_n-\rho_B)E_B$$

$$2U_B=(1_n+\rho_B)\phi(1_n-\rho_A)E_A+(1_n+Q\rho_B^{-1})(1_n-\rho_B)E_B$$

$$\rho_A=(Z_A-Z_0)(Z_A+Z_0)^{-1}$$

$$\rho_B=(Z_B-Z_0)(Z_B+Z_0)^{-1}$$

$$Z_0=X\ \mathrm{diag}(\ \sqrt{(R_i+sL_i)/sC_i}\ )X^t$$

$$\phi=X\ \mathrm{diag}[\exp(-\sqrt{(R_i+sL_i)sC_i}\ \ell)]X^{-1}$$

Table 2.Coupled and Decoupled Transmission-Line parameters

(a) coupled-line parameters:

$$L=\begin{bmatrix} 3 & 1 & 0.5 \\ 1 & 4 & 1 \\ 0.5 & 1 & 3 \end{bmatrix}\ \mathrm{nH/cm},\quad R=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}\ \Omega/\mathrm{cm}$$

$$C=\begin{bmatrix} 44 & -10 & -4 \\ -10 & 35 & -1 \\ -4 & -10 & 44 \end{bmatrix}\ \mathrm{pF/cm},\quad \ell=10\sqrt{2}\ \mathrm{cm},\quad \nu=\sqrt{2}\,10^{10}\ \mathrm{cm/sec}.$$

(b) decouple-line parameters:

$$X=\begin{bmatrix} 0.3 & 0.5 & 0.8 \\ -1.2 & 0 & 0.8 \\ 0.3 & -0.5 & 0.8 \end{bmatrix}$$

$$(\bar{C}_1,\bar{C}_2,\bar{C}_3)=(\ 3\ ,\ 1,\ \ 2\ )\ \mathrm{pF}$$

$$(\bar{R}_1,\bar{R}_2,\bar{R}_3)=(20/9,\ 4,\ 5/4\ )\ \ \Omega$$

$$(\bar{L}_1,\bar{L}_2,\bar{L}_3)=(\ 5/3,\ 5,\ 5/2\ )\ \mathrm{nH}$$

(c)Pade synthesis of lossy characteristic impedances

$$(r_1,r_2,r_3)=(23.57,70.71,\ 35.35)\ \Omega$$
$$(c_1,c_2,c_3)=(63.64,35.35,113.14)\ \mathrm{pF}$$

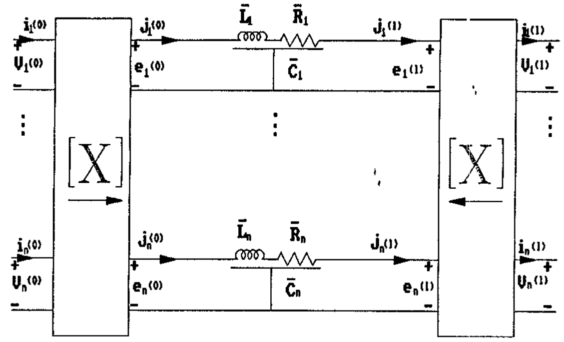

Fig.1 (b) the decoupled equivalent circuit

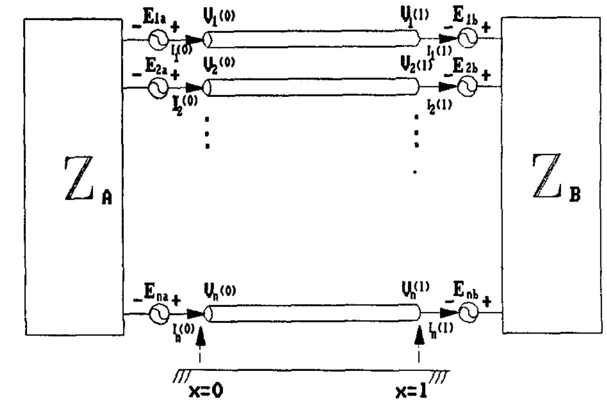

Fig.2(a)Thevenin's termination for derivation of waveform relaxation algorithm

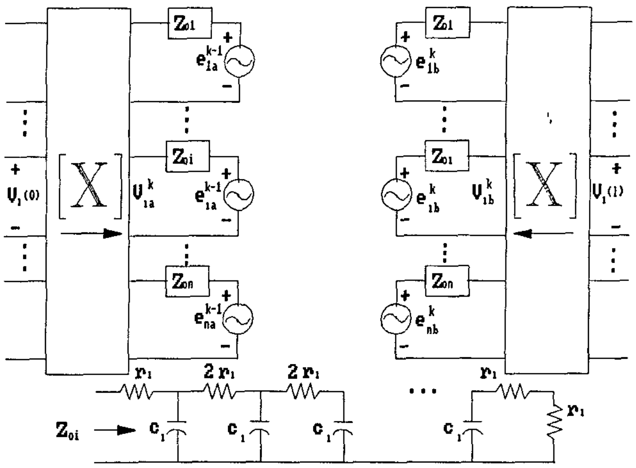

Fig.2 (b) Disjoint 2-part network for iterative waveform relaxation

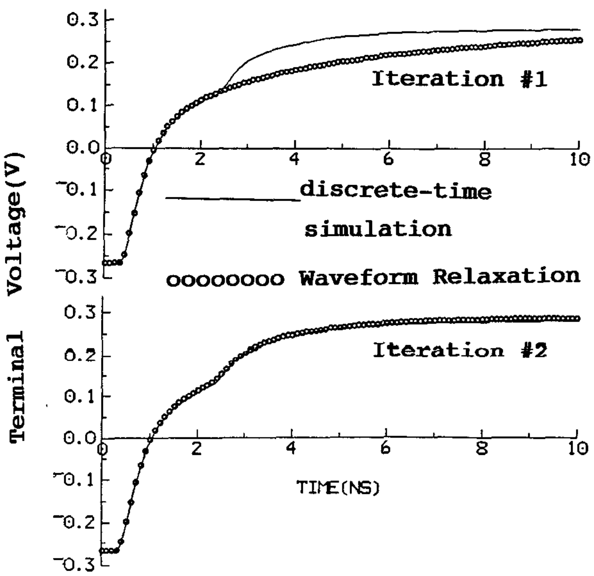

Fig.3 (b) Transient response at the near-end of the center-conductor

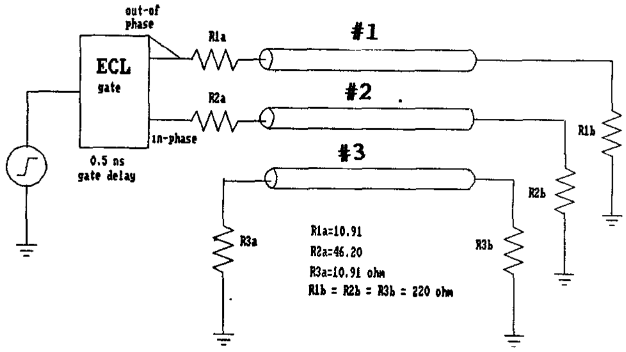

Fig.3(a)A three-conductor system driven by a bipolar ECL-gate

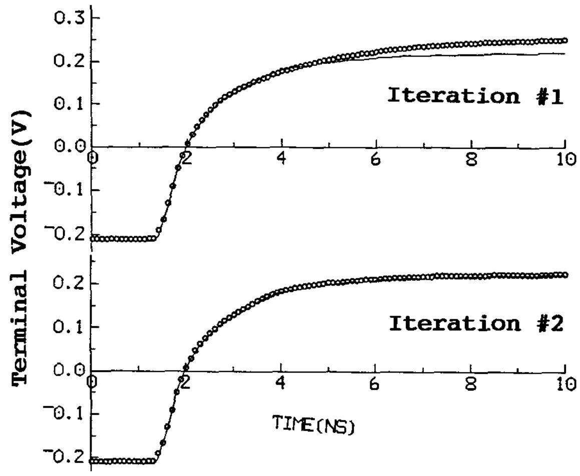

Fig.3(c) Transient response at the far-end of the center-conductor

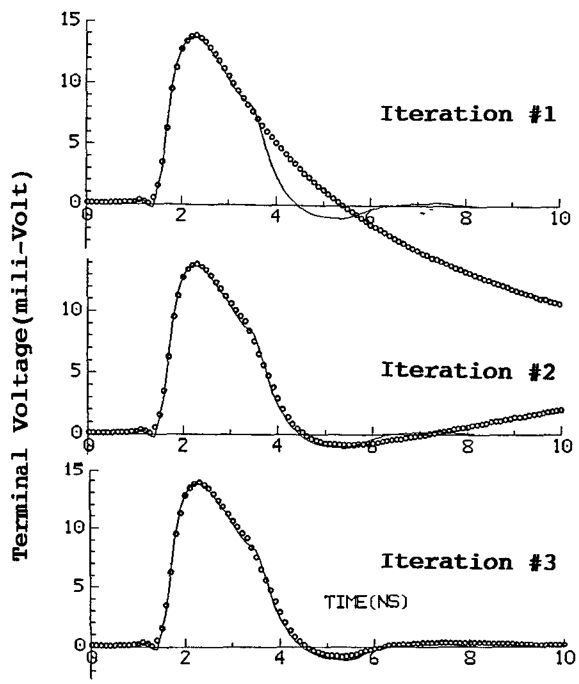

Fig.3(d) Transient response at the far-end of the quiet-conductor